Stochastic optimization algorithms
Lecture 16, 20181010

Problem-solving and review

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NOTE!

- This week, there is no lecture on Friday!
- Thus, the next lecture is on Tuesday (20181016).
- However, we are of course reachable via e-mail (and in our offices) if you have questions regarding HP2.

Exam information

- An exam kit (three practice exams) is available on the web page. Exam contents:
  - Questions of understanding. For example: *How does tournament selection work? Define and describe the penalty term used in the Penalty method* etc. etc.
  - Computational problems. Old exam problems: 2.5, 2.9, 2.10, 2.11, 2.13, 3.2, 3.6, 3.8, 3.9.
  - Proofs. See Appendix B. For example: B1.1, B2.4 etc. Any proof covered during the lectures may appear on the exam. (A complete list will be given later).
  - There will be *no* Matlab questions on the exam.

Today's learning goals

- After this lecture you should be able to
  - Solve a set of exam problems

Old exam problem (1)

2. Consider a function adaptation task in which linear genetic programming (LGP) is used for finding an unknown function $f(x)$ based on measurements taken for several different values of x. The LGP chromosomes consist of a sequence of instructions, each represented using four genes. The first gene in each instruction represents the operator, the second gene reprents the destination register, and the two remaining genes are the operands. In this task, there are three variable registers (denoted r_1, r_2, and r_3), and three constant registers (denoted c_1, c_2, and c_3). There are four operators, namely o_1 (addition), o_2 (subtraction), o_3 (multiplication) and o_4 (division). Initially the constant registers are set as $c_1 = 1$, $c_2 = 2$ and $c_3 = -1$. The variable registers are initiated as $r_1 = x$, $r_2 = r_3 = 0$. The output (i.e. the estimate $\hat{f}(x)$) is taken as the contents of r_2. The operands are chosen from the set $\{a_1, \ldots a_6\} = \{r_1, r_2, r_3, c_1, c_2, c_3\}$.

   (a) Consider an LGP chromosome given by

   $$c_1 = 1214\,3315\,3123\,3333\,1323\,4213. \qquad (2)$$

   Which function is obtained when decoding this chromosome? (2p)

   (b) During mutation, the fourth gene in the chromosome c_1 is mutated from 4 to 1. What will be the corresponding function? (1p)

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Old exam problem (2)

3. Ant colony optimization (ACO), which is inspired by the behavior of ants, is typically used for solving routing problems, such as the traveling salesman problem (TSP).

   (a) Consider the construction graph (for TSP) shown in Fig. 1 (see next page). If the level of artificial pheromone is equal to 0.5 for all edges e_{ij}, what is the probability that an ant will follow the nearest neighbour path, starting from node 1? For the parameter values, choose $\alpha = 1$ and $\beta = 2$. Make sure to include all relevant intermediate steps in your calculations! (3p)

   (b) Several ACO algorithms have been defined, one of which is the Max-min ant system (MMAS) in which (among other things) pheromone limits are imposed. Consider again the construction graph for TSP in Fig. 1. Assuming that MMAS is being used, with $N = 4$ artificial ants, and that the initial pheromone levels τ_{ij} in this case are equal to $1/(\rho D_{\text{nn}})$ for all edges e_{ij}, where D_{nn} is the length of the nearest-neighbour path starting from node 1 (i.e. the path considered above), determine the pheromone levels for *all* edges e_{ij} after the first iteration, where the four ants followed the paths $(1,4,2,5,3)$, $(2,4,1,3,5)$, $(5,3,1,4,2)$, and $(1,2,3,4,5)$, respectively. (Note that, as usual, the ants also return to their start node in the final step.) For the pheromone updating rule, set the evaporation rate ρ to 0.5, and the pheromone limits $\tau_{\min}$ and $\tau_{\max}$ to 0.1 and $1/(\rho D_{\text{nn}})$, respectively. (4p)

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Old exam problem (2)

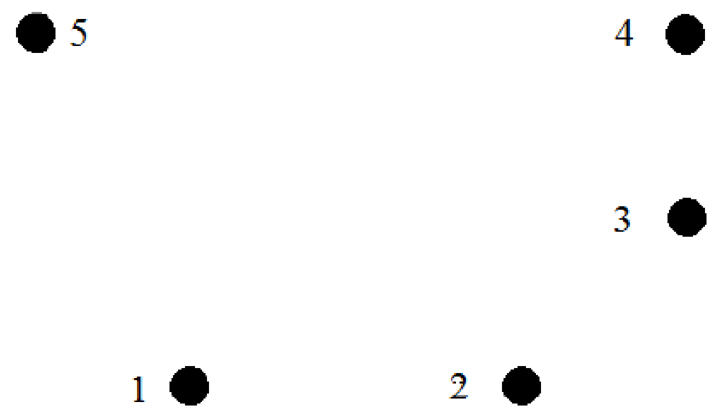

Figure 1: Construction graph for Problem 3. The nodes are located at (1, 0) (node 1), (3, 0) (node 2), (4, 1) (node 3), (4, 2) (node 4), and (0, 2) (node 5).

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Old exam problem (3)

4. The penalty method is a classical optimization method (which, however, also can be used in connection with stochastic optimization) for solving constrained minimization problems.

   (a) In the penalty method a penalty function is used for measuring the degree to which the constraints are violated for a given variable vector $\mathbf{x}$. The penalty function is then added to the objective function $f(\mathbf{x})$. Write down the general expression for the penalty function, carefully explaining all variables and parameters. (1p)

   (b) Use the penalty method to find the minimum of the function

$$f(x_1, x_2) = (x_1 - 6)^2 + (x_2 - 7)^2, \tag{3}$$

subject to the constraints

$$g_1(x_1, x_2) = -3x_1 - 2x_2 + 6 \leq 0, \tag{4}$$

$$g_2(x_1, x_2) = -x_1 + x_2 - 3 \leq 0, \tag{5}$$

$$g_3(x_1, x_2) = x_1 + x_2 - 7 \leq 0, \tag{6}$$

and

$$g_4(x_1, x_2) = \frac{2}{3}x_1 - x_2 - \frac{4}{3} \leq 0, \tag{7}$$

Hint: Start at the unconstrained minimum, and examine the constraints carefully. It is also a good idea to plot the set of feasible points before starting with the actual minimization. (4p)

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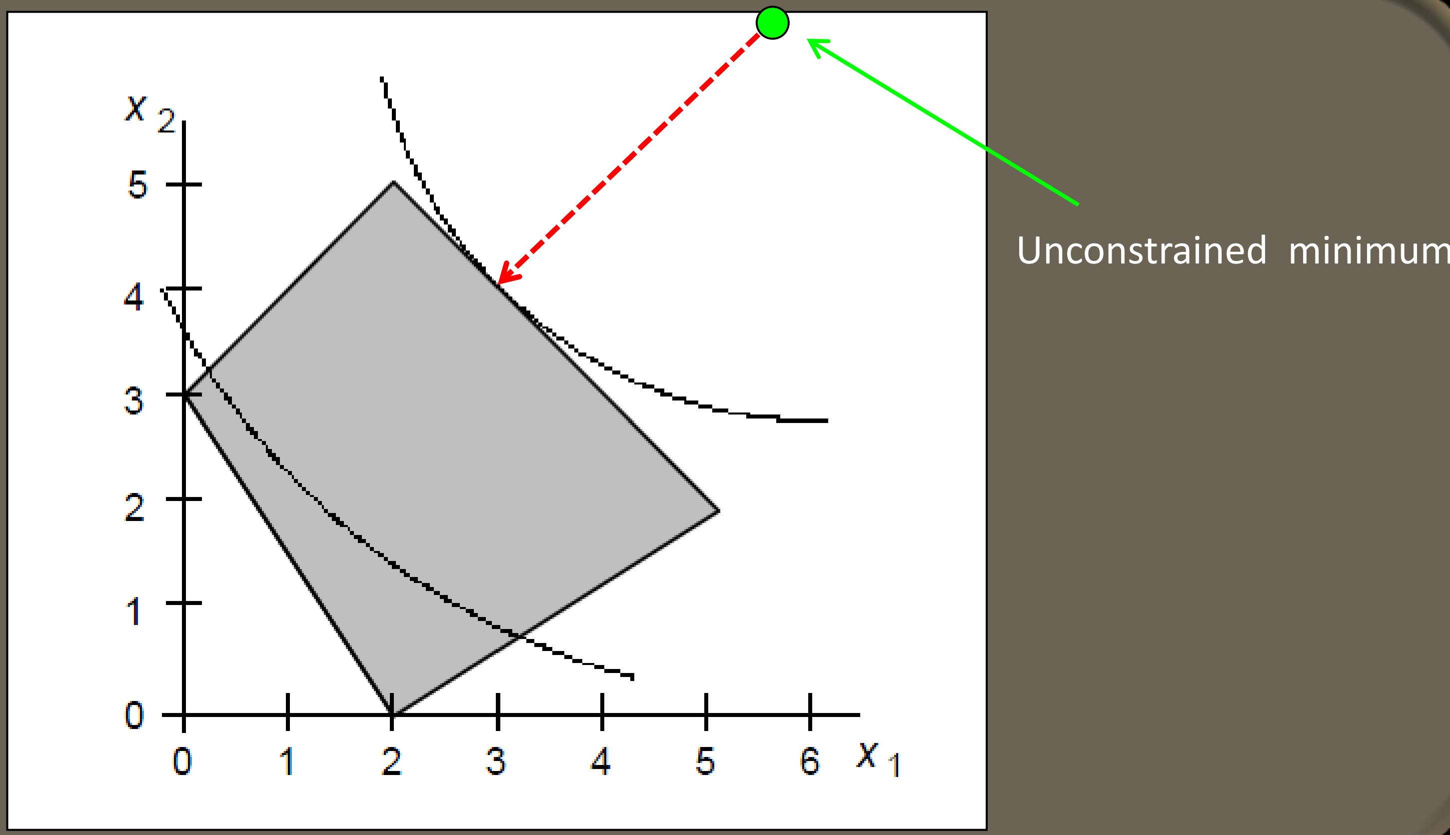
Old exam problem (3)



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Deadlines for hand-ins

- Deadline for HP2: **20181017** (to avoid point deductions).
- Strict, final deadline for handing in the *first attempt* of HP1 and HP2: **20181030;** see also the course memo (available on the web page).
- Revisions can be handed in later, though, but they should be handed in as soon as possible.