Autonomous agents
Lecture 3, 20160125

Kinematics and dynamics of wheeled robots

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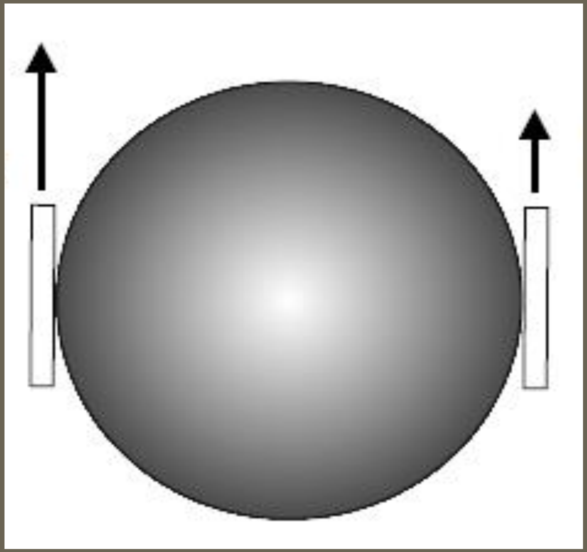
Today's learning goals

- After this lecture you should be able to
  - Derive the kinematic equations for a two-wheeled differentially steered robot.
  - Describe the concept of odometric drift, and explain why independent calibration is needed.
  - Derive the dynamic equations for a two-wheeled differentially steered robot.

Differentially steered robots

- Here, we will consider only differentially steered two-wheeled robots (with a third point for support).
- Differentially steered robot: Two independently controlled wheels (one motor for each).



Kinematics

- The study of the robot's possible motions, without consideration of forces.

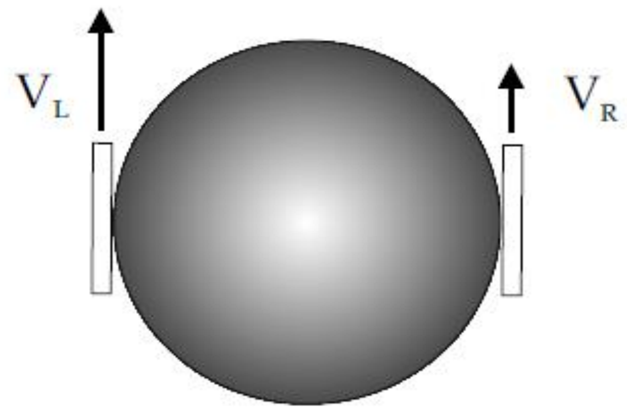


Figure 2.1: *A schematic representation of a two-wheeled, differentially steered robot.*

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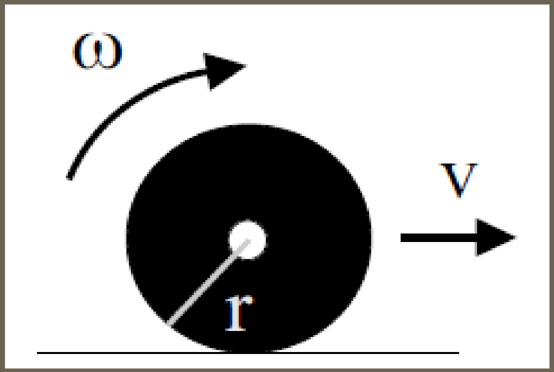
Kinematics

- Assumption: No slipping, i.e. the wheels are just rolling, without sliding.



- In this case, for a wheel with radius r, the following holds:

$$v = \omega r$$

- … where v is the linear speed of the wheel and ω its angular speed.

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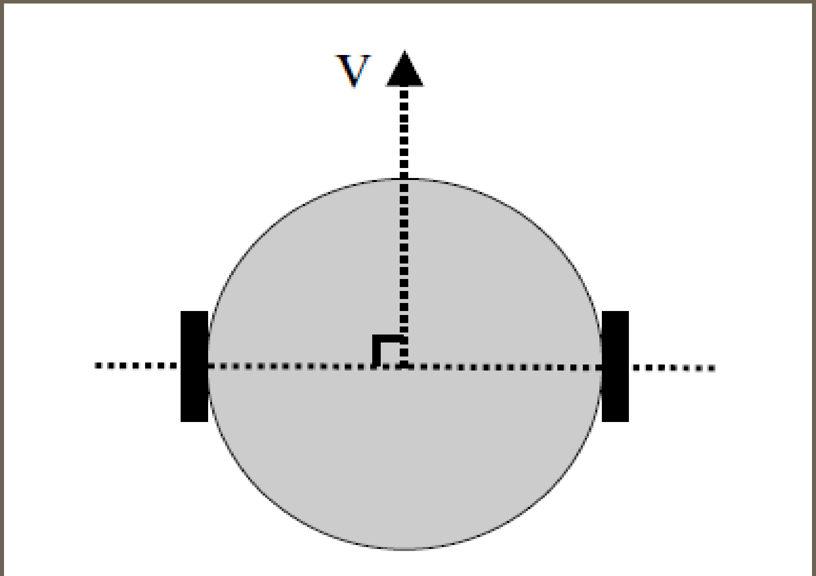
Kinematics

- Moreover, the instantaneous direction of motion (φ) will be perpendicular to the wheel axes:



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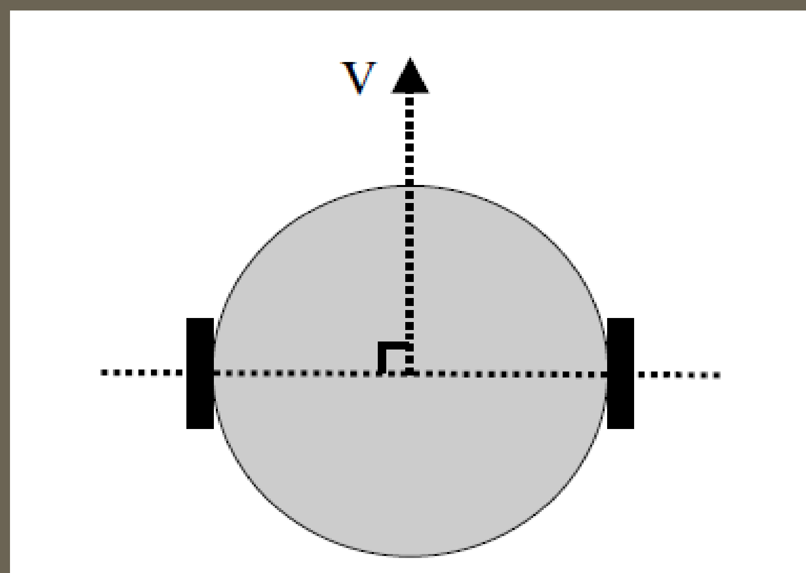
Kinematics

- Forward kinematics: Finding (x, y, φ), given v_L and v_R.



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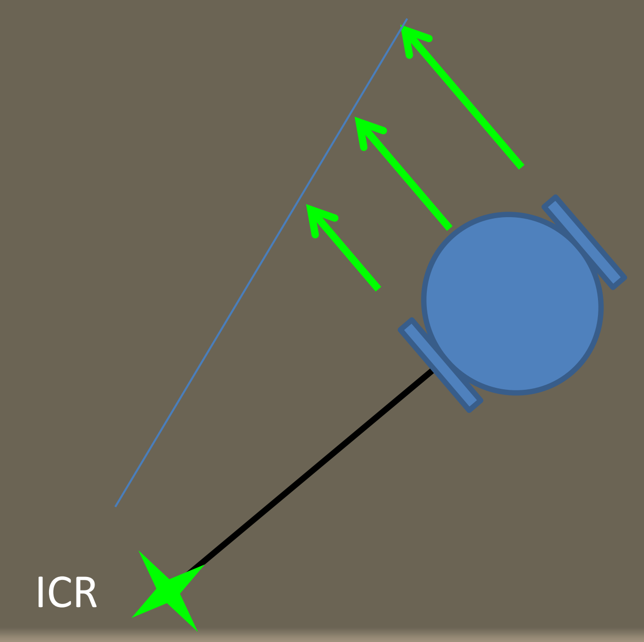
Instantaneous center of rotation

- The point (not attached to the robot!) relative to which the motion of the robot is a pure rotation.



Kinematic equations

- Derivation: pp. 22-23 in the lecture notes.
- Final equations:

$$
\begin{aligned}
X(t_1) - X_0 &= \int_{t_0}^{t_1} V_x(t)\mathrm{d}t = \int_{t_0}^{t_1} \frac{v_L(t) + v_R(t)}{2} \cos\varphi(t)\mathrm{d}t, \\
Y(t_1) - Y_0 &= \int_{t_0}^{t_1} V_y(t)\mathrm{d}t = \int_{t_0}^{t_1} \frac{v_L(t) + v_R(t)}{2} \sin\varphi(t)\mathrm{d}t, \\
\varphi(t_1) - \varphi_0 &= \int_{t_0}^{t_1} \omega(t)\mathrm{d}t = -\int_{t_0}^{t_1} \frac{v_L(t) - v_R(t)}{2R}\mathrm{d}t,
\end{aligned}
$$

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Odometry and localization

- Odometry (here): Inferring the robot's pose (position and heading) based on integration of wheel rotation.
- Factors that limit odometric accuracy
  - In practice, summation rather than integration, i.e. dt is replaced by Δt (not infinitesimal)
  - Limited number of pulses per revolution (limitation of the wheel encoders)
  - Wheel slip
  - Inaccurate kinematic model (no wheels are exactly circular, no wheels axes are exactly straight etc etc.)

Odometry and localization

- Thus, there will be an odometric error, which will grow with time, as the robot moves.
- In order to get an accurate (over time) pose estimate, an independent method for odometric calibration is needed.
- This topic (localization) will be considered later in the course.

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Dynamics

- The study of a the motion resulting from the forces acting on the robot.
- Basically an application of Newton's 2nd law, in linear form

$$\boldsymbol{F} = m\,\ddot{\boldsymbol{x}}$$

…and in angular form

$$\tau = \bar{I}\ddot{\varphi}$$

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Dynamics

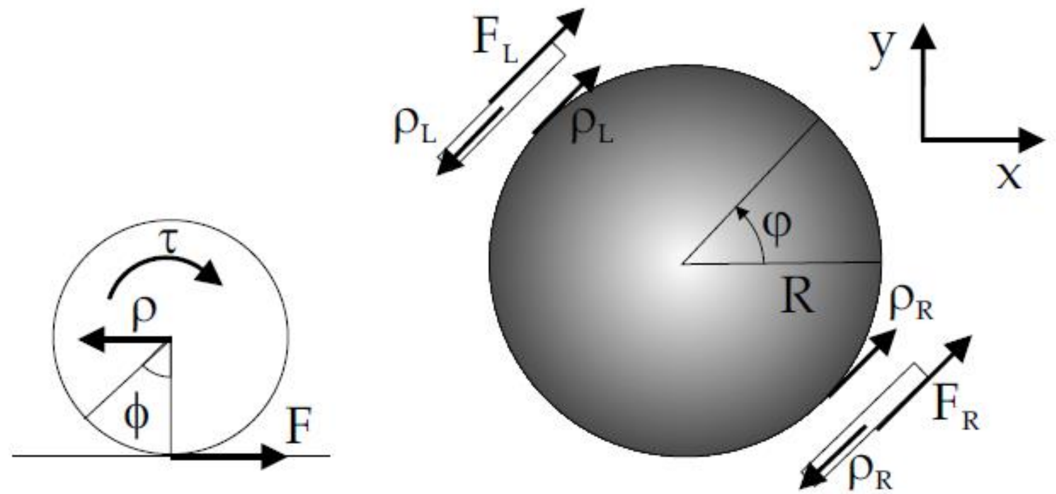

Figure 2.3: *Left panel: Free-body diagram showing one of the two wheels of the robot. Right panel: Free-body diagram for the body of the robot and for the two wheels. Only the forces acting in the horizontal plane are shown.*

Dynamics

- Derivation: pp. 24-27 in the lecture notes.
- Final equations

$$M\dot{V} + \alpha V = A(\tau_L + \tau_R)$$

$$I\ddot{\varphi} + \beta\dot{\varphi} = B(-\tau_L + \tau_R)$$

- …where $A = 1/r$ and $B = R/r$.

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