Chalmers University of Technology
Department of Applied Mechanics

Examples of exam problems in FFR 125 (Autonomous Agents)

It is allowed to use a calculator, as long as it cannot store any text. It is also allowed to use tables such as Standard Math, Beta etc. (provided that no notes have been added). It is *not* allowed to use the lecture notes, slides, or any other papers from the course during the exam.

Note! In problems involving computation, show *clearly* how you arrived at your answer, i.e. include all relevant intermediate steps etc. Only giving the answer will result in zero points on the problem in question.

In the actual exam (on 2014-03-11) there will be four or five problems, and the maximum number of points will be equal to 25.

Note that the questions below are *examples*. The problems given in the actual exam may concern any topic covered in the lecture notes.

1. (a) During navigation, mobile robots generally keep track of their position using *odometry*. Describe this procedure in detail, including also a description of sensors used when generating odometric estimates. (2p)

   (b) In order for a robot to maintain an accurate estimate of its pose over large distances, odometry alone is not sufficient: An independent method for localization (odometric recalibration) is needed to counteract the inevitable odometric drift. Describe, in as much detail as possible, a localization method for indoor robot applications. (2p).

   (c) The use of the utility concept (in economics, robotics etc.) can be motivated by means of the *St. Petersburg paradox*. Describe this paradox, as well as the way to resolve it using the concept of utility. (Make sure to give the relevant equations). (2p)

   (d) Autonomous robots commonly use some form of direct-current (DC) motor for their movements. Describe the relevant parts of a standard DC motor (*not* servo motor). Also, describe briefly how such a motor works. (1p)

   (e) The electrical equations for a DC motor can be derived noting that the circuit representing the motor can be described as a resistor in series with an inductance and the back EMF (counteracting the applied voltage V). Noting that the back EMF is proportional to the angular velocity, and neglecting electrical transients, derive an expression for the generated torque (for a given voltage V) as a function of the angular velocity of the motor shaft. (2p)

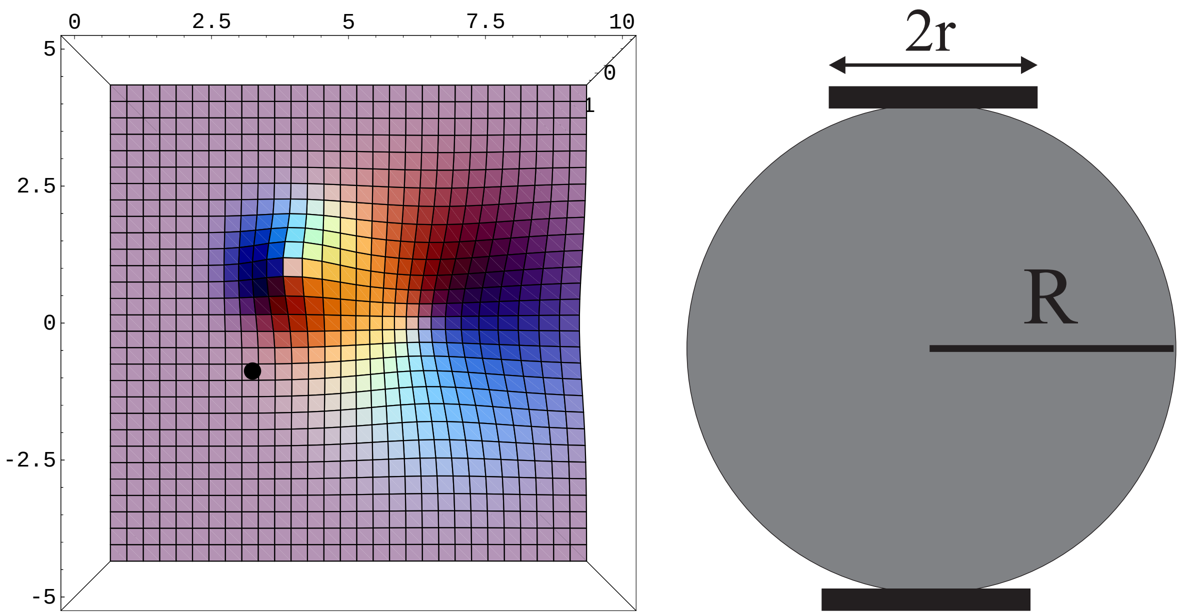

Figure 1: Left panel: The potential field from Problem 2. Right panel: The robot considered in Problem 3.

2. Mobile robot navigation can be achieved using many different methods, of which potential field navigation is an example. Consider the navigation problem illustrated in the left panel of Fig. 1 in which a robot, illustrated as a black disc, is supposed to move towards a target (shown as a valley) located at $(x_t, y_t) = (7, 0)$. The arena also contains an obstacle at $(x_o, y_o) = (4, 1)$, which appears as a hill in the figure. The potentials of the goal and the obstacle are both given by

$$\phi(x, y; x_p, y_p, \alpha, \beta, \gamma) = \alpha e^{-\left(\frac{x-x_p}{\beta}\right)^2 - \left(\frac{y-y_p}{\gamma}\right)^2}, \tag{1}$$

where $\alpha = -2$, $\beta = \gamma = 2$ for the target and $\alpha = \beta = \gamma = 1$ for the obstacle.

(a) Without inserting any of the numerical parameters given above, write down the combined potential $\Phi(x, y)$ generated by the target and the obstacle. Next, derive an expression for the suggested direction of heading (obtained from the potential field) $\hat{\mathbf{r}}$ at an arbitrary point (x, y) for this potential field. The heading vector $\hat{\mathbf{r}}$ is a unit vector. You should explicitly write down the detailed expression for the numerator of $\hat{\mathbf{r}}$, but you do not need to write down the denominator (i.e. the normalizing factor) explicitly. (2p)

(b) Assuming that the robot starts at $(x, y) = (3, -1)$ determine the direction of heading suggested by the potential field. (1p)

(c) Next, without considering kinematics and dynamics, let the robot move one length unit in the direction determined in part (b). Which point is reached, and what is the direction of heading suggested by the potential field at this point? (2p)

3. Consider the two-wheeled differentially steered robot shown (from above) in the right panel of Fig. 1. The radius of the robot's body is denoted R and its mass (excluding the wheels) is denoted M. The mass of each wheel is equal to m and the wheel radius is r. The torques (from the motors) acting on the left and right wheels are denoted τ_{L} and τ_{R}, respectively. The heading angle of the robot is denoted φ. The moment of inertia of the robot's body (with respect to a vertical axis through its center of mass) is denoted $\overline{I}$, and the moments of inertia of the two wheels are denoted $\overline{I}_{\mathrm{w}}$. Furthermore, let ϕ_{L} and ϕ_{R} denote the rotation angle of the left and right wheel, respectively. You may assume that the wheels roll without slipping.

   (a) Generate a free-body diagram, in which the body of the robot and the two wheels are considered separately. In your diagram, indicate all relevant forces and torques. Next, write down the dynamic equations for the two wheels and for the robot's body (six equations in total). (3p)

   (b) Write down (and explain) the kinematic equations for the two wheels and for the body of the robot (four equations in total). (1p)

   (c) Neglecting damping (and other losses) and assuming $m \ll M$, $\overline{I}_{\mathrm{w}} \ll \overline{I}$, and $r \ll R$, derive (and simplify as much as possible) the equations of motion for the robot, i.e. the equations determining the variation (with time) of V (the robot's speed) and φ. (3p)

4. The theory of rational decision-making uses the concept of *utility* as a common currency for the comparison of different potential outcomes, and thus as a guide for action selection.

   (a) Consider a case in which one aims to determine the utility of money for a person A, who has assigned utilities such that $u(0) = 0$ (the utility of gaining 0 dollars) and $u(100) = 100$ (the utility of gaining a hundred dollars). Furthermore, A is indifferent between (1) an outcome c_1 in which she would receive 30 dollars (with certainty) and (2) a situation in which she would receive either 0 dollars (with 50% probability) or 100 dollars, with the same probability. Determine the utility of c_1 (for person A). Show clearly how you arrive at your answer. (1p).

   (b) Continuing the previous problem, consider now also a person B whose point of indifference instead occurs at 60 dollars, i.e. he would be indifferent between (1) the outcome c_2 of receiving 60 dollars (with certainty) and outcome (2), as described in (b). (B assigns the same utilities, $u(0)$ and $u(100)$, as A). What can be said concerning the attitudes toward risk of persons A and B? Motivate your answer clearly! (1p)

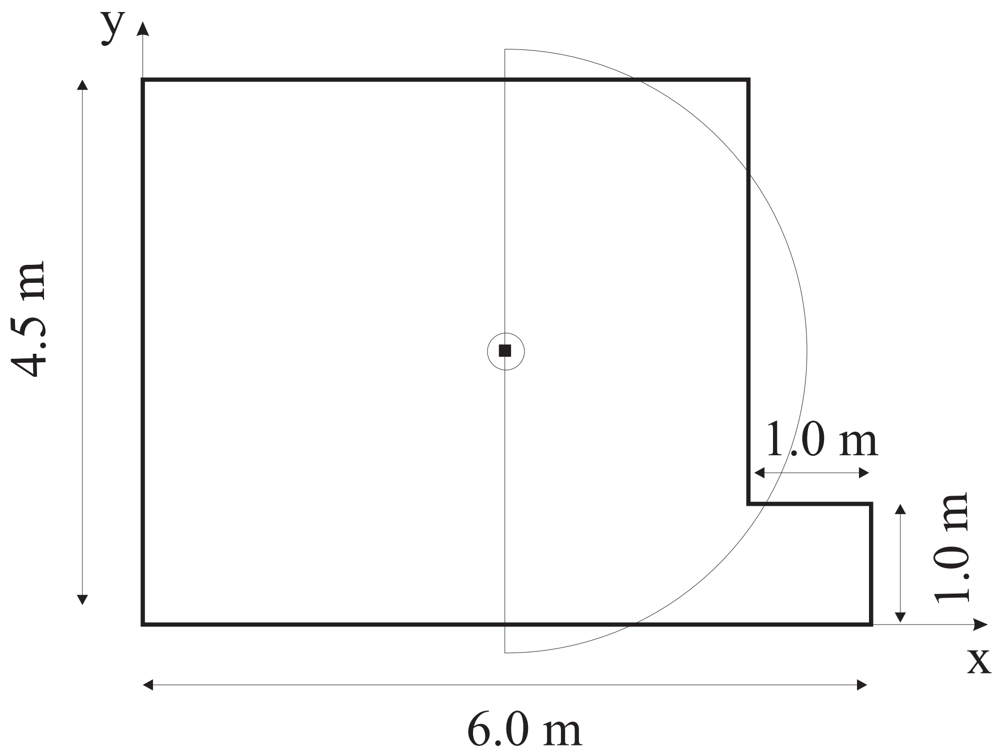

Figure 2: Arena for problems 5(b) and 5(c). The robot is located at $(3.00, 2.25)$.

5. Autonomous robots are commonly equipped with wheel encoders, using which the rotation of the wheels can be determined so that, in turn, the pose of the robot can be estimated with the help of a kinematic model of the robot. However, odometry alone is not sufficient to maintain an accurate pose estimate over large distances, because of the odometric drift. Thus, an independent method for localization (i.e. odometric recalibration) is needed. In the course, a method for odometric recalibration, based on laser scan matching, has been studied.

   (a) Describe this method in detail, defining clearly the concepts of the virtual laser range finder, scan matching (including relevant equations), as well as the search procedure used for improving the scan match. (3p)

   (b) Consider the situation shown in Fig. 2 in which a robot is placed in a simple arena consisting of six wall segments that are aligned with the coordinate axes. The origin of the coordinate system is located in the lower left corner of the figure. The robot is located at the point $(3.00, 2.25)$, and its heading angle φ is equal to 0. A laser range finder (LRF) is mounted on top of the robot, at the center of its circular body. The scan region (neglecting the obstacles) of the LRF (which has a range of 2.5 m and an opening angle of 180 degrees) is indicated in the figure. Assuming that only the readings in five directions (relative to the robot's angle of heading) are considered, namely the directions $-\pi/2, -\pi/4, 0, \pi/4$ and $\pi/2$, determine the ray readings in the situation shown in the figure. (1p)

Note: You do *not* need to derive and use general equations for line-line intersection. Instead, you can make use of the simple geometry of the arena, and the fact that the robot's direction of heading coincides with the x axis, when computing ray lengths.

(c) Due to odometric drift the robot's *estimated* position will normally differ from the true position. Consider a case in which the robot's estimated position is $(3.50, 2.00)$, and the estimated heading is 0, i.e. equal to the true heading. Assuming that the map is available and accurately represents the true arena, determine (i) the ray readings for the virtual LRF (reading from the map, at the estimated pose), for the five rays defined in (b), and (ii) the scan match error (defined by the equation used in the localization method described in (a)) between the actual LRF reading (computed in (b)) and the virtual reading. (2p)